Using mastery teaching in Maths to raise attainment at KS4 and beyond

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Aims of the session

1. Understand what is meant by ‘Mastery’
2. To understand the teaching principles that promote Mastery in Mathematics
3. Mastery and raising attainment - the challenge!!
There are $n$ sweets in a bag. 6 of the sweets are orange. The rest of the sweets are yellow.

Hannah takes at random a sweet from the bag. She eats the sweet.

Hannah then takes at random another sweet from the bag. She eats the sweet.

The probability that Hannah eats two orange sweets is $\frac{1}{3}$

(a) Show that $n^2 - n - 90 = 0$

My mum works for an accountancy company and it took 4 accountants 2hrs to answer the sweets Q. They have maths degrees. #EdexcelMaths

The probability of me getting a good grade in this exam is 1/1000. How many sweets does this mean I have? #EdexcelMaths
JOURNEY STARTS IN SHANGHAI

TIME TO RESEARCH - DESIRE TO IMPROVE

OBSERVATION ROOM

OBSERVERS
What is mastery?

“A man cannot understand the art he is studying if he only looks for the end result without taking the time to delve deeply into the reasoning of the study.”

— Miyamoto Musashi (1584-1645) - an expert Japanese swordsman
What is mastery?

If you drive a car, imagine the process you went through...

- The very first drive, lacking the knowledge of what to do to get moving
- The practice, gaining confidence that you are able to drive
- The driving test, fairly competent but maybe not fully confident
- A few years on, it’s automatic, you don’t have to think about how to change gears or use the brake
- Later still, you could teach someone else how to drive

Learning to master driving takes time and a lot of practice!
So where are you on this journey?
Some over-arching features...

• A belief that the vast majority of students can achieve in line with national expectations.

• Differentiation – NOT through new content but through scaffolding, questioning and rapid intervention

• AfL is crucial and is present throughout the lesson and used to direct the direction of the lesson.
Some over-arching features...

• Sufficient **TIME** is given to ensure deep understanding – but how?? **Less spiral more depth and greater links.**

• Additional practice **outside of class**
  – Apps and puzzles in form time/out of class settings.
  – Regular short homework to consolidate class work
  – **BUILD RESILIENCE** and CHECK UNDERSTANDING
Tom and Mary share some money in the ratio 3 : 2. Tom gets £12, how much does Mary get?

Concrete

Tom is 3 parts and £12
One part: 12 ÷ 3 = £4
Mary is 2 parts
Mary: 4 x 2 = £8
Mary has £8

Pictorial

draw bar model showing ratio 3:2 and Tom getting £12
find 1 part is £4
Mary gets £8
Efficiency and reasoning
- explore, reason and find a strategy

Calculate: \( \frac{2}{3} \times \left( \frac{5}{6} + \frac{3}{7} \right) \)

Can you solve this problem in any other method? Which one do you think is more simple?
What do you find?

\[
\frac{2}{3} \times \left( \frac{5}{6} + \frac{3}{7} \right)
\]

\[
= \frac{2}{3} \times \left( \frac{5 \times 7 + 3 \times 6}{6 \times 7} \right)
\]

\[
= \frac{2}{3} \times \frac{35 + 18}{42}
\]

\[
= \frac{2}{3} \times \frac{53}{42}
\]

\[
= \frac{53}{63}
\]
The flow diagram of the addition and subtraction of fractions

1. Input
2. Check if the denominators are the same.
   - Yes: Proceed to Calculate.
   - No: Proceed to Reduce.
3. Calculate
4. Reduce
5. Output
what is 45 as a product of prime factors?

“45 can be written as a product of prime factors as 3 x 3 x 5 or 3² x 5”

NOT

“3 x 3 x 5” or “3² x 5”
Apply knowledge of solving equations in order to spot misconceptions and correct them.

**JUDGE** - Which answer is correct? Justify your answer. Can you spot the misconceptions?

Liam looks at the picture below, makes an equation and solves it. Which option shows the correct equation and solution?

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>[5x + 50 = 250]</td>
<td>[5x = 50 + 250]</td>
<td>[5x = 300]</td>
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<tr>
<td>[x = 60]</td>
<td>[x = 50]</td>
<td>[x = 70]</td>
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<td>[250 - 5x = 50]</td>
<td>[x = 40]</td>
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Precise language
- used by students AND teachers

\[
\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}
\]

Used from Year 1 by teachers AND students

\[
\frac{8}{15} \div \frac{4}{5} = \frac{2}{3}
\]

\[
\frac{8}{15} \div \frac{2}{3} = \frac{4}{5}
\]

Cannot be simplified because 4 and 5 are CO-PRIME

Factors

\[
a \times b = c
\]

\[
c \div b = a \quad c \div a = b
\]
Commutativity
- understanding keywords/key concepts can make a big difference

<table>
<thead>
<tr>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<tbody>
<tr>
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<td>$1 \times 1 = 1$</td>
<td>$2 \times 2 = 4$</td>
<td>$3 \times 3 = 9$</td>
<td>$4 \times 4 = 16$</td>
<td>$5 \times 5 = 25$</td>
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<td>$1 \times 2 = 2$</td>
<td>$2 \times 3 = 6$</td>
<td>$3 \times 4 = 12$</td>
<td>$4 \times 5 = 20$</td>
<td>$5 \times 6 = 30$</td>
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<td>$2 \times 4 = 8$</td>
<td>$3 \times 5 = 15$</td>
<td>$4 \times 6 = 24$</td>
<td>$5 \times 7 = 35$</td>
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<td>4</td>
<td>$1 \times 4 = 4$</td>
<td>$2 \times 5 = 10$</td>
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<td>$3 \times 7 = 21$</td>
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<td>$3 \times 8 = 24$</td>
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<td>$1 \times 8 = 8$</td>
<td>$2 \times 9 = 18$</td>
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<td>9</td>
<td>$1 \times 9 = 9$</td>
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Symbolic representation
- generalises learning and shows deeper understanding

Describe the basic property of fractions

A fraction remains the same value, when both its numerator and denominator are multiplied or divided by the same non-zero number.

\[
\frac{a}{b} = \frac{a \times k}{b \times k} = \frac{a \div n}{b \div n}
\]

\( (b \neq 0, k \neq 0, n \neq 0) \)
Interim methods
- stepping stones that are visited for a short period of time

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<th>U</th>
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<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>

\[
3x + 4 = 7
\]

\[
x = 1
\]

\[
\begin{align*}
3x + 4 &= 7 \\
x &= 3 \\
\div 3 &= 1
\end{align*}
\]

\[
\begin{align*}
3x + 4 &= 7 \\
-4 &= 3 \\
\div 3 &= 1
\end{align*}
\]
Variation

- Present the learning in different ways to deepen the understanding

Procedural Variation

• Varying the conditions, results, generalities:
  – Same problem but varying the numbers.
  – Same problem but varying the unknowns.
  – Same structure and numbers but varying the context.

• Varying the method to solve the problem

• Varying the application of the method

Conceptual Variation

• Same problem but varying the representation.
If I want to divide 3 pies into 4 equal parts. How many pies will be in each part?

7
Magic Squares

```
  7
25 10 13
30
24 18
16 22 20 10
26 6
```

Ink Blots/
Missing Digit

```
24 +
| 3
```

Darts?

```
```

Cryptarithms/
Alphametics

```
AB
+ AB
-----
BC
```
(two answers)
Example of how a BIDMAS lesson could look
Making Links

– Concrete to Abstract
– Across topic areas to promote logical progression
– Between topic areas to see how the Maths links together
Link secure known knowledge to new abstract content....

If I know the area of a rectangle is base \( x \) height

Then I know the area of this rectangle is either....

\[
A = 5x + 2x
\]
\[
A = (5 + 2) \times x
\]
\[
A = 7x
\]

Therefore....

\[
5x + 2x = 7x \text{ (collecting like terms)}
\]
Can you combine two congruent triangles to make a parallelogram?

Think: what's the relationship between each area of a triangle with the area of a parallelogram.

Each area of a triangle is the half area of the parallelogram.
Intelligent Practice Promotes Reasoning

- Exercises are carefully varied
- Content is developed in small steps
- Fluency and conceptual knowledge embedded
- **QUALITY** not quantity!
Calculate the area of a parallelogram

4.5 cm \times 3 = 13.5 \text{ cm}^2

4 \times 3.5 = 14 \text{ cm}^2

12 \times 8 = 96 \text{ cm}^2

9.6 \times 10 = 96 \text{ cm}^2
Calculate

<table>
<thead>
<tr>
<th>parallelogram</th>
<th>base (cm)</th>
<th>height (cm)</th>
<th>area ($cm^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>8</td>
<td>80</td>
</tr>
<tr>
<td></td>
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<td>4</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>14</td>
<td>42</td>
</tr>
</tbody>
</table>

\[ A = b \times h \]

\[ b = \frac{A}{h} \]

\[ h = \frac{A}{b} \]
If two parallelograms have equal bases and equal heights, their areas are both same.
Choose

CD = ? Which calculation is correct? (C)

(A) $5 \times 4 \div 3$  (B) $3 \times 4 \div 5$

(C) $5 \times 3 \div 4$  (D) $5 \times 3 \times 4$
True or false

• If the areas of two parallelograms are the same, they must have equal bases and equal heights. (×)

• If a triangle and a parallelogram have equal bases and equal heights, the area of the parallelogram must be two times of that of the triangle. (√)
How can mastery raise attainment...

The Challenges

– Processes vs. understanding
– KS4 is too late
– Depth of understanding from day 1
– Mastery scheme of work at primary with little or no support
– The new specifications – depth vs. content
How can mastery raise attainment...

Where to start...

– Quick wins - techniques I’ve shown today
– mastery schemes of work that start in Year 7 – less spiral
– Dialogue between primary and secondary schools
– We must support primary Maths coordinators.
– Use your Maths Hubs.
What can it achieve???

\[
125 \times 80.8 = 125 \times (80 + 0.8) = 125 \times 80 + 125 \times 0.8 = 10000 + 100 = 10100
\]

\[
125 \times 80.8 = 125 \times 8 \times 10.1 = 1000 \times 10.1 = 10100
\]

Students prefer 2nd method.
Thank you for listening

Any questions?

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