



St John the Baptist School
Have faith.. believe you can



"The whole is greater than the sum of its parts."
Aristotle

Using mastery teaching in Maths to raise attainment at KS4 and beyond

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Aims of the session

1. Understand what is meant by 'Mastery'
2. To understand the teaching principles that promote Mastery in Mathematics
3. Mastery and raising attainment - the challenge!!



GCSE maths students vent fury over exam question about sweets they claim was 'too hard' **Mirror**

- 19 There are n sweets in a bag.
6 of the sweets are orange.
The rest of the sweets are yellow.
- Hannah takes at random a sweet from the bag.
She eats the sweet.
- Hannah then takes at random another sweet from the bag.
She eats the sweet.
- The probability that Hannah eats two orange sweets is $\frac{1}{3}$
- (a) Show that $n^2 - n - 90 = 0$

More than 5,000 people have signed a petition urging exam board Edexcel to lower its grading boundaries following a question about sweets

What [#EdexcelMaths](#) were thinking whilst making our exam paper

← ↻ ★ 👤 ...

My mum works for an accountancy company and it took 4 accountants 2hrs to answer the sweets Q. They have maths degrees. [#EdexcelMaths](#)

The probability of me getting a good grade in this exam is 1/1000. How many sweets does this mean I have? [#EdexcelMaths](#)



Twitter

TIME TO RESEARCH - DESIRE TO IMPROVE



OBSERVATION ROOM



OBSERVERS

What is mastery?

“A man cannot understand the art he is studying if he only looks for the end result without taking the time to delve deeply into the reasoning of the study.”

— *Miyamoto Musashi (1584-1645) - an expert Japanese swordsman*



What is mastery?

If you drive a car, imagine the process you went through...



- The very first drive, lacking the knowledge of what to do to get moving
- The practice, gaining confidence that you are able to drive
- The driving test, fairly competent but maybe not fully confident
- A few years on, it's automatic, you don't have to think about how to change gears or use the brake
- Later still, you could teach someone else how to drive

Learning to master driving takes time and a lot of practice!



So where are you on this journey?



Some over-arching features...

- A belief that the **vast majority** of students **can achieve** in line with national expectations.
- **Differentiation** – NOT through new content but through **scaffolding**, **questioning** and **rapid intervention**
- **AfL** is crucial and is present throughout the lesson and used to **direct the direction** of the lesson.



Some over-arching features...

- Sufficient **TIME** is given to ensure deep understanding – but how?? **Less spiral more depth and greater links.**
- Additional practice **outside of class**
 - Apps and puzzles in form time/out of class settings.
 - Regular short homework to consolidate class work
 - BUILD RESILIENCE and CHECK UNDERSTANDING

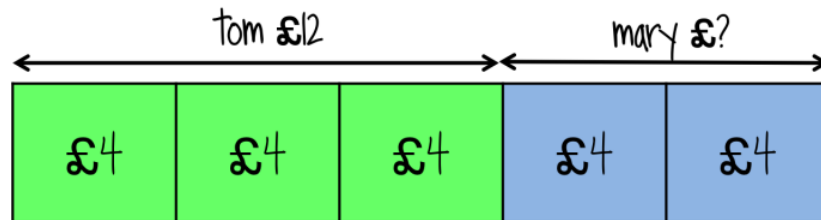
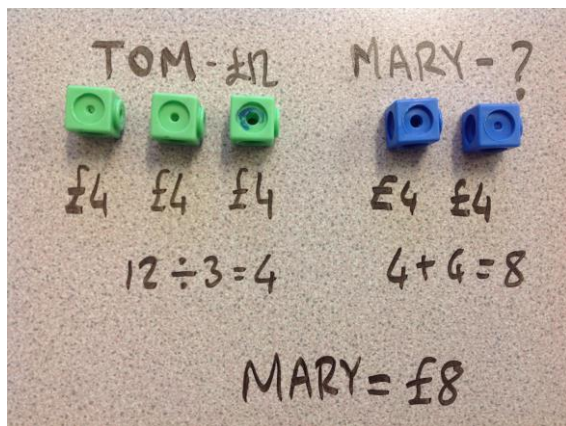


Concrete – Pictorial - Abstract

1

Tom and Mary share some money in the ratio 3 : 2. Tom gets £12, how much does Mary get?

Concrete



draw bar model showing ratio 3: 2 and tom getting £12
find 1 part is £4
mary gets £8

Pictorial

Abstract

Tom is 3 parts and £12
One part: $12 \div 3 = £4$
Mary is 2 parts
Mary: $4 \times 2 = £8$
Mary has £8



Efficiency and reasoning

- explore, reason and find a strategy

2

Calculate: $\frac{2}{3} \times (\frac{5}{6} + \frac{3}{7})$

Can you solve this problem in any other method? which one do you think is more simple?



What do you find?

2

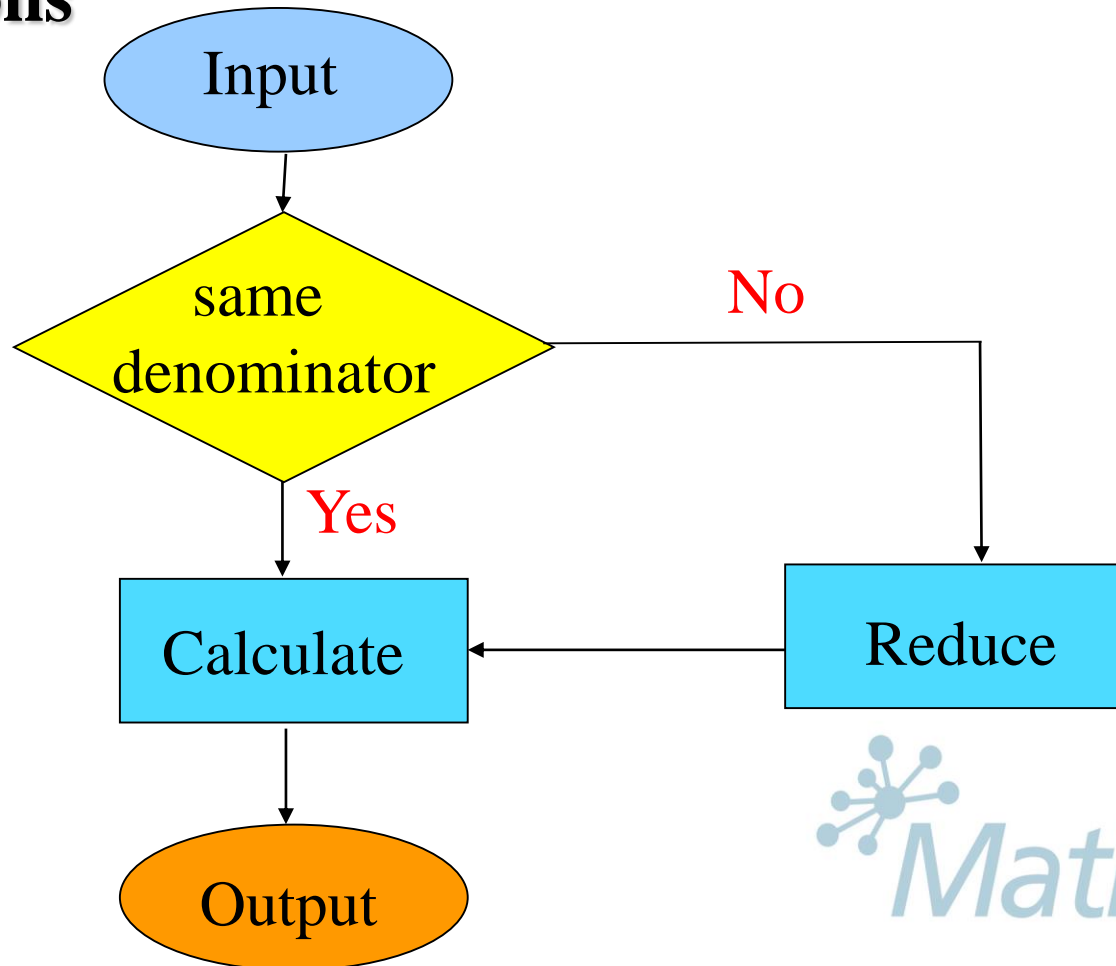
$$\begin{array}{l} \frac{2}{3} \times \left(\frac{5}{6} + \frac{3}{7} \right) \\ \swarrow \quad \searrow \\ = \frac{2}{3} \times \left(\frac{5}{6} + \frac{3}{7} \right) \qquad = \frac{2}{3} \times \frac{5}{6} + \frac{2}{3} \times \frac{3}{7} \\ = \frac{2}{3} \times \frac{35+18}{42} \qquad = \frac{35+18}{63} \\ = \frac{53}{63} \qquad = \frac{53}{63} \end{array}$$



Strategy

2

The flow diagram of the addition and subtraction of fractions



Students give conclusions

3

- TALK MATHS

What is 45 as a product of prime factors?

“45 can be written as a product of prime factors
as $3 \times 3 \times 5$ or $3^2 \times 5$ ”

NOT

“ $3 \times 3 \times 5$ ” or “ $3^2 \times 5$ ”

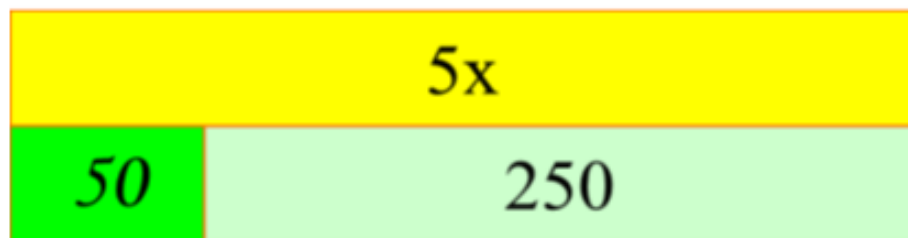


Apply knowledge of solving equations in order to spot misconceptions and correct them

3

JUDGE - Which answer is correct? Justify your answer.
Can you spot the misconceptions?

Liam looks at the picture below, makes an equation and solves it.
Which option shows the correct equation and solution?



A

$$\begin{aligned} 5x + 50 &= 250 \\ x &= 60 \end{aligned}$$

B

$$\begin{aligned} 5x &= 50 + 250 \\ x &= 50 \end{aligned}$$

C

$$\begin{aligned} 5x &= 300 \\ x &= 70 \end{aligned}$$

D

$$\begin{aligned} 250 - 5x &= 50 \\ x &= 40 \end{aligned}$$

Precise language

- used by students **AND** teachers

4

$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

factor factor product

$$a \times b = c$$

$$c \div b = a \quad c \div a = b$$

product factor factor

Used from Year 1 by teachers AND students

$$\frac{8}{15} \div \frac{4}{5} = \frac{2}{3}$$

$$\frac{8}{15} \div \frac{2}{3} = \frac{4}{5}$$

Cannot be simplified because 4 and 5 are **CO-PRIME**



Commutativity

- understanding keywords/key concepts can make a big difference

4

1	2	3	4	5
$1 \times 1 = 1$	$2 \times 2 = 4$	$3 \times 3 = 9$	$4 \times 4 = 16$	$5 \times 5 = 25$
$1 \times 2 = 2$	$2 \times 3 = 6$	$3 \times 4 = 12$	$4 \times 5 = 20$	$5 \times 6 = 30$
$1 \times 3 = 3$	$2 \times 4 = 8$	$3 \times 5 = 15$	$4 \times 6 = 24$	$5 \times 7 = 35$
$1 \times 4 = 4$	$2 \times 5 = 10$	$3 \times 6 = 18$	$4 \times 7 = 28$	$5 \times 8 = 40$
$1 \times 5 = 5$	$2 \times 6 = 12$	$3 \times 7 = 21$	$4 \times 8 = 32$	$5 \times 9 = 45$
$1 \times 6 = 6$	$2 \times 7 = 14$	$3 \times 8 = 24$	$4 \times 9 = 36$	
$1 \times 7 = 7$	$2 \times 8 = 16$	$3 \times 9 = 27$		
$1 \times 8 = 8$	$2 \times 9 = 18$			
$1 \times 9 = 9$				

6	7	8	9
$6 \times 6 = 36$	$7 \times 7 = 49$	$8 \times 8 = 64$	$9 \times 9 = 81$
$6 \times 7 = 42$	$7 \times 8 = 56$	$8 \times 9 = 72$	
$6 \times 8 = 48$	$7 \times 9 = 63$		
$6 \times 9 = 54$			



Symbolic representation

- generalises learning and shows deeper understanding

5

Describe the basic property of fractions

**A fraction remains the same value,
when both its numerator and denominator
are multiplied or divided by the same
non-zero number.**

$$\frac{a}{b} = \frac{a \times k}{b \times k} = \frac{a \div n}{b \div n}$$

$$(b \neq 0, k \neq 0, n \neq 0)$$



Interim methods

6

- stepping stones that are visited for a **short** period of time

	H	T	U	
		2	5	
x			7	
		3	5	
	1	4	0	
	1	7	5	

	H	T	U	
		2	5	
x			7	
	1	7	5	
		3		



$$3x + 4 = 7$$

$$\begin{array}{ccccccc} x & \xrightarrow{\quad} & x3 & \xrightarrow{\quad} & +4 & \xrightarrow{\quad} & 7 \\ 1 & \xleftarrow{\quad} & \div 3 & \xleftarrow{\quad} & -4 & \xleftarrow{\quad} & 7 \end{array}$$

$$x = 1$$



$$\begin{array}{rcl} 3x + 4 = 7 & & \\ -4 \downarrow & & -4 \\ 3x = 3 & & \\ \div 3 \downarrow & & \div 3 \\ x = 1 & & \end{array}$$



Variation

7

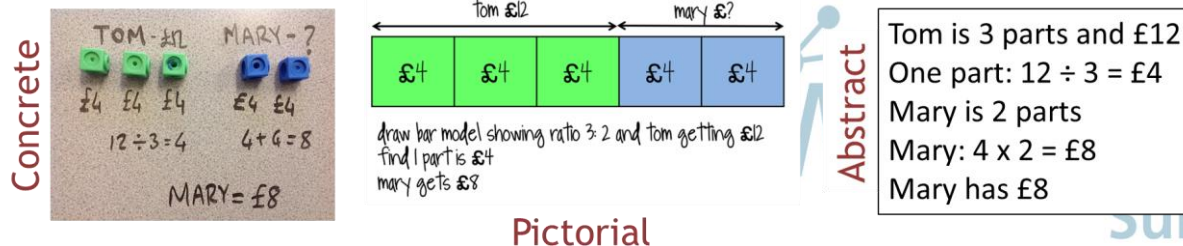
- Present the learning in different ways to deepen the understanding

Procedural Variation

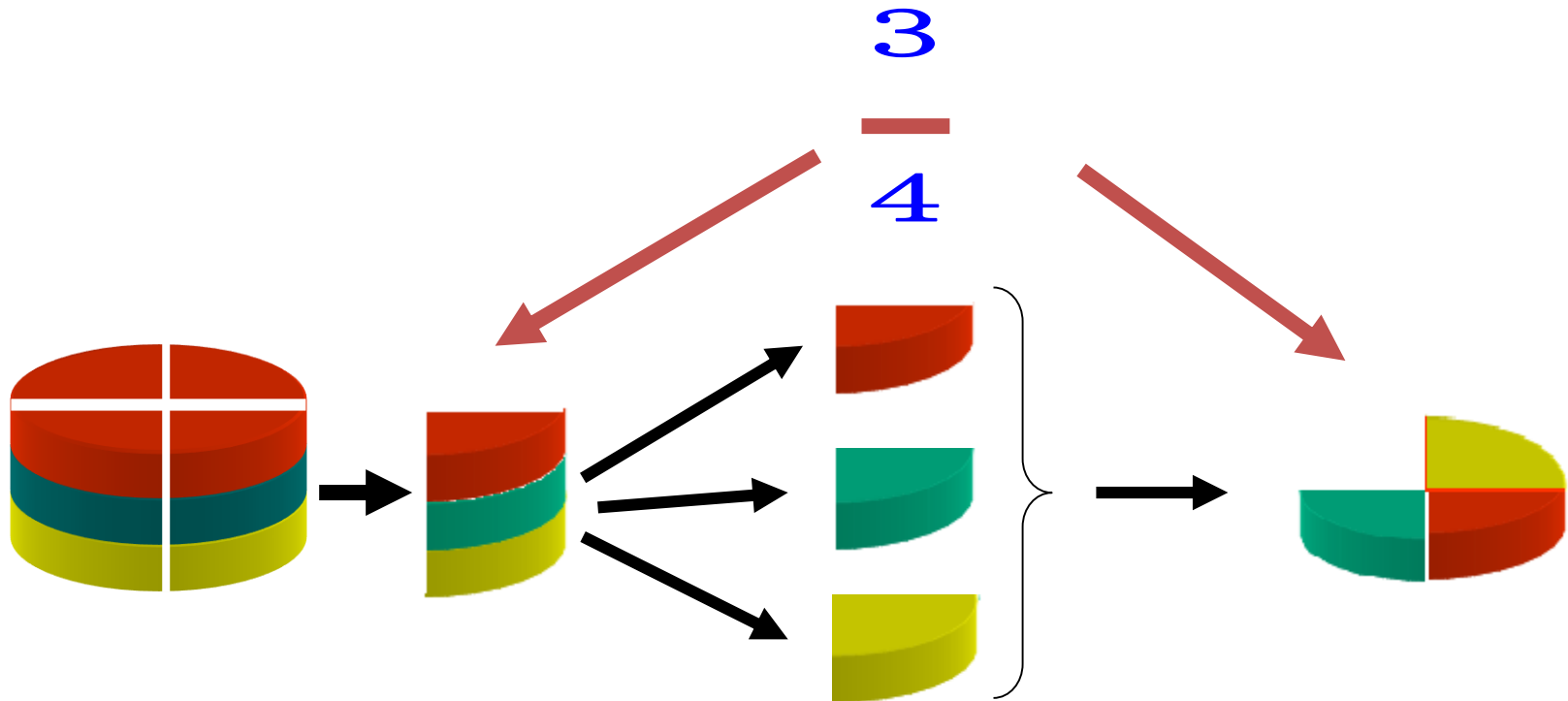
- Varying the conditions, results, generalities:
 - Same problem but varying the numbers.
 - Same problem but varying the unknowns.
 - Same structure and numbers but varying the context.
- Varying the method to solve the problem
- Varying the application of the method

Conceptual Variation

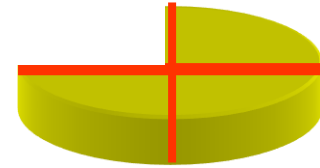
- Same problem but varying the representation.



*If I want to divide 3 pies into 4 equal parts.
How many pies will be in each part?*



7



Magic Squares

		7
25	10	13

		30	
24			18
16	22	20	10
26		6	

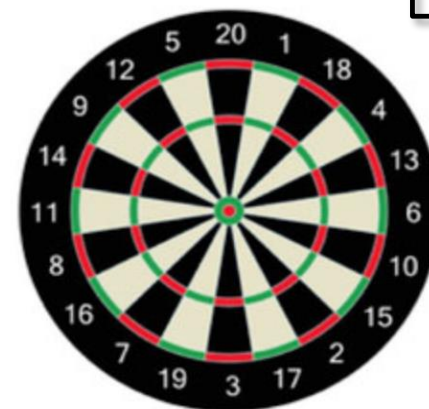
Ink Blots/ Missing Digit

$$\begin{array}{r} 2 \\ 24 \\ + \\ 3 \end{array}$$

		T	U	
		2	★	
	+	▲	2	
		3	5	

		H	T	U	
		▲	7		
	+		5	★	
		1	3	5	

Darts?



Cryptarithms/ Alphametics

$$\begin{array}{r} AB \\ AB \\ + \\ \hline BC \end{array}$$

(two answers)



An adaption of a lesson that I have taught for years:

7

BIDMAS

00:02 00

Show all your working!

BIDMAS

00:02 00

Show all your working!

BIDMAS - Resilience test!!

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.
- 7.
- 8.
- 9.
- 10.

Write out these questions. Some may be correct.

★ ★

1). $3 + 1 \times 5 = 20$

4). $12 - 6 \times 4 = 24$

7). $5 \times 9 - 7 = 10$

★ ★ Fill in the boxes using the number at the end of each row.

Example. (1, 2, 3, 4, 5)

1). (1, 2, 3, 4, 5)

3). (1, 2, 3, 4, 5)

5). (1, 2, 3, 4, 5)

7). (1, 2, 3, 4, 5)

Extens

At least 5 questions

	÷		+		8
×		-		×	
	+		+		15
-		×		+	
	+		+		12
38		1		61	

Can you arrange all of the digits from 1 to 9 in each of empty boxes above? You can only use each number once. Remember the rules of BIDMAS.



Example of how a BIDMAS lesson could look

Making Links

8

- Concrete to Abstract
- Across topic areas to promote logical progression
- Between topic areas to see how the Maths links together



Link secure known knowledge to new abstract content....

8

If I know the area of a rectangle is base x height

Then I know the area of this rectangle is either....

$$A = 5 \times x + 2 \times x$$

$$A = 5x + 2x$$

OR

$$A = (5 + 2) \times x$$

$$A = 7 \times x$$

$$A = 7x$$

x cm

5 cm

2 cm



Therefore....

$$5x + 2x = 7x \text{ (collecting like terms)}$$

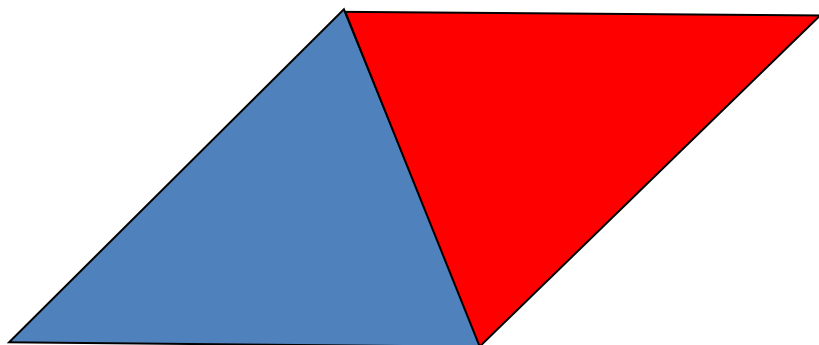


Links across a topic for progression....

8

Can you combine two congruent triangles to make a parallelogram?

Think: what's the relationship between each area of a triangle with the area of a parallelogram.



Each area of a triangle is the **half** area of the parallelogram.



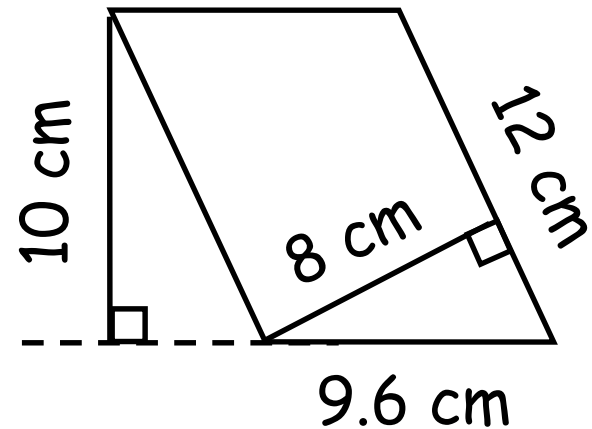
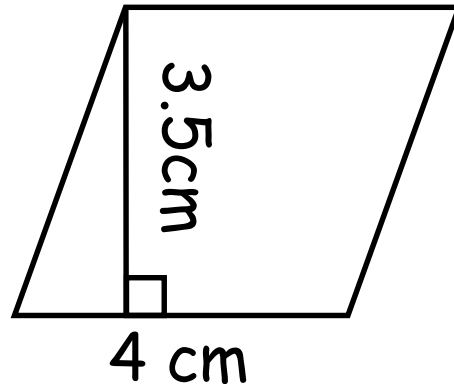
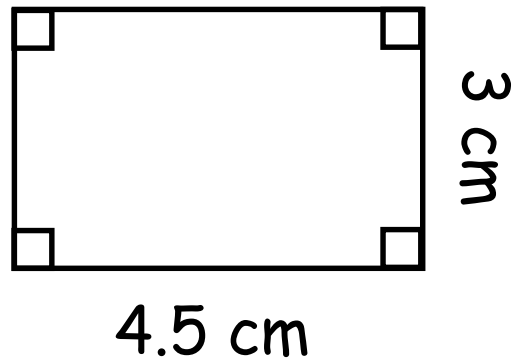
Intelligent Practice Promotes Reasoning

- Exercises are carefully varied
- Content is developed in small steps
- Fluency and conceptual knowledge embedded
- **QUALITY** not quantity!



Variation in area of parallelograms exercises.....

Calculate the area of a parallelogram



$$4.5 \times 3 = 13.5 \text{ (cm}^2\text{)} \quad 4 \times 3.5 = 14 \text{ (cm}^2\text{)} \quad 12 \times 8 = 96 \text{ (cm}^2\text{)}$$

$$9.6 \times 10 = 96 \text{ (cm}^2\text{)}$$



Calculate

parallelogram	base	10cm	25m	3cm
	height	8cm	4m	14cm
	area	80cm ²	100m ²	42cm ²

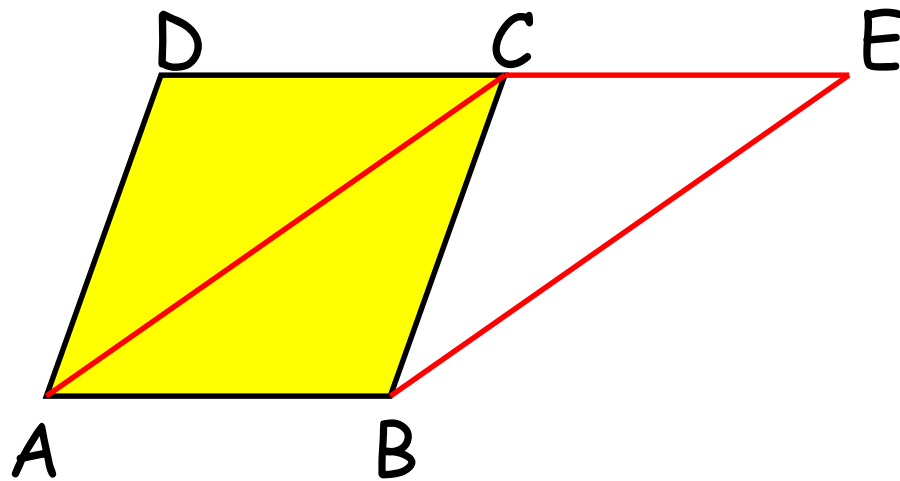
$$A = b \times h \rightarrow$$

$$b = A \div h$$

$$h = A \div b$$



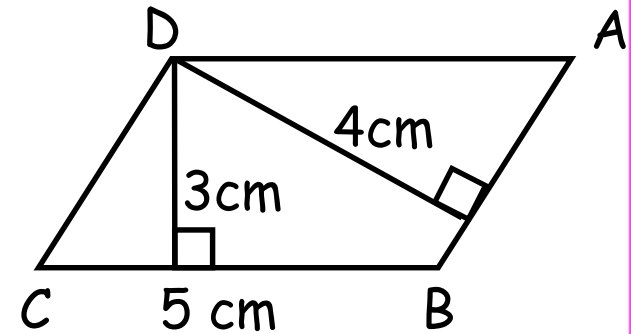
$$A_{\square ABCD} = A_{\square ABEC} ?$$



If two parallelograms have equal bases and equal heights, their areas are both same.

Choose

$CD = ?$ Which calculation is correct? (**C**)



(A) $5 \times 4 \div 3$ (B) $3 \times 4 \div 5$

(C) $5 \times 3 \div 4$ (D) $5 \times 3 \times 4$



True or false

- If the areas of two parallelograms are the same, they must have equal bases and equal heights. (\times)
- If a triangle and a parallelogram have equal bases and equal heights, the area of the parallelogram must be two times of that of the triangle .(\checkmark)



How can mastery raise attainment...

The Challenges

- Processes vs. understanding
- KS4 is too late
- Depth of understanding from day 1
- Mastery scheme of work at primary with little or no support
- The new specifications – depth vs. content



How can mastery raise attainment...

Where to start...

- Quick wins - techniques I've shown today
- mastery schemes of work that start in Year 7 – less spiral
- Dialogue between primary and secondary schools
- We must support primary Maths coordinators.
- Use your Maths Hubs.



What can it achieve???

$$\begin{aligned}125 \times 80.8 \\&= 125 \times (80 + 0.8) \\&= 125 \times 80 + 125 \times 0.8 \\&= 10000 + 100 \\&= 10100\end{aligned}$$

$$\begin{aligned}125 \times 80.8 \\&= 125 \times 8 \times 10.1 \\&= 1000 \times 10.1 \\&= 10100\end{aligned}$$

Students prefer 2nd method.



Thank you for listening

Any questions?

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